Analysis for Water Hammer considering the effect of Fluid Structure Interaction in Straight Pipes

A THESIS SUBMITTED IN THE PARTIAL FULFILLMENTS OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY

In

MECHANICAL ENGINEERING

By

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Roll number 108ME050

Department of Mechanical Engineering
National Institute of Technology, Rourkela
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Under the Guidance of

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C E R T I F I C A T E

This is to certify that the work in this thesis entitled "**Analysis for Water Hammer considering the effect of Fluid Structure Interaction in Straight Pipes"** by **Suyash Mishra (108ME050),** has been carried out under my supervision in partial fulfillment of the requirements for the degree of **Bachelor of Technology** in ***Mechanical Engineering*** during session 2011- 2012 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

Date 10th May, 2012
Place- Rourkela

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CONTENTS

ABSTRACT

Water hammer is a serious problem in flow pipelines of nuclear power plant as fluid structure interaction significantly affects the response of pipe components and performance of such plants as a whole. This phenomenon is basically observed when there is sudden increase in flow velocity, abrupt change in cross section and if the flow is forced fully to stop. The purpose of this study is to use a finite element approach for studying such a highly nonlinear problem in order to investigate the effect of fluid structure interaction (FSI) in pipe lines. In this study, the transient water hammer problem is attempted by considering the effect of all three Poisson, friction and junction couplings. The continuity and momentum equations are written in terms of pressure and velocity of the flow. Along with this the pipe-structural equations in axial direction is considered. Compared to conventional water hammer relations, present work accounts the Poisson coupling as an important term in the fluid dynamic equations. The complete interaction of fluid and pipe comes with boundary condition, which is taken as gradual valve closure. There are various solution approaches such as method of characteristics (MOC), finite element-MOC mixed method as well as finite element approach alone. In present work, finite element approach is employed to mesh the fluid and solid regions. In FEM method two kind of formulation has been done to account for symmetric matrix formulation and shape function used in determining the mass stiffness and damping matrix element. This kind of formulation uses velocity potential (ϕ) and hydrostatic pressure (p) as unknowns in fluid domain and axial displacement (u) as unknown in solid domain. A MATLAB code has been written to calculate result for Finite element formulation by solving the nonlinear second-order differential equations using Newmark-beta scheme.

LIST OF FIGURE

NOMENCLATURE

E- Young modulus (N/m^2)

K_f -Bulk Modulus (N/m^2)

D-Diameter (m)

e- Thickness (m)

f- Friction factor

L-Length (m)

C_0-Initial wave Velocity (m/sec^2)

M_s- Mass of pipe (Kg/m)

M_f-Mass of fluid (Kg/m)

P- Density (Kg/m^3)

F_s—Force due to fluid on structure

F_f-Force due to structure on fluid

F_v-Force due to Valve

Cv- Valve closure coefficient

ρ_f- Density of fluid(Kg/m^3)

ρ_s- Density of solid(pipe) (Kg/m^3)

σ- Axial stress in pipe (N/m^2)

ν- Poisson ratio

ε- Strain Tensor

Φ- Velocity Potential

P- Pressure

A_c-Area of cross section (m^2)

A_s- Area of Solid(Pipe) (m^2)

A_f- Area of Pipe (m^2)

Chapter 1

INTRODUCTION

1.1 Problem Statement

Study of water hammer phenomenon has become very important not only in terms of safety reason but also from the economics point of view as any mishap can result to low performance of plant. Various attempt like analytical modeling, computational modeling and experimental modeling has been made to get a greater insight of the problem but still it is most challenging problem to deal with because of its highly non linear nature and coupling between two domain of physics solid and fluid.

Water Hammer is phenomenon of sudden rise of pressure due to closing of valve or due to any other similar cause and subsequent propagating of wave along the pipe causing noise and excitation. The rise of pressure may ever burst the pipe. The magnitude of pressure depends upon speed of valve closure, velocity of flow, length of pipe, elasticity of pipe and flowing fluid.

A classical water hammer approach has been established which describe the phenomenon in more general way and predicts correctly extreme pressure and velocity of the wave but this theory could not predict or measure damping and sudden change in wave traversing along the pipe. The reason behind above failure of theory is non-consideration of different coupling mechanism like friction, Poisson and junction coupling which governs the exact nature of water hammer phenomenon.

The waves exert acoustic pressure on the solid and the solid and fluid are coupled because of this force exerted by fluid flow on solid. This fluid flow results in deformation of structure and deformation of structure cause the change in fluid flow. This coupled deformation of fluid and solid is term as fluid structure interaction. All the three coupling play a major role in above mechanism. Water hammer phenomenon is caused by number of phenomenon mentioned below:

1. Pressure wave in single –phase flow
2. Water column separation
3. Flow into voided section
4. Acceleration of a water slug in steam lines
5. Steam condensation induced water hammer
6. Two phase transient
7. Valve Dynamics/ Instabilities

In all above major reason for water hammer in plant setup we will confine our self in dealing with valve dynamics / instabilities problem for single phase flow which is also termed as Delft Hydraulics Benchmark Problem.

The FSI has got great attention in recent years because of safety issue, reliability of plant set up, environmental concern in pipe delivery system and plant performance. The Fluid Structure interaction process basically deals with transfer of momentum and forces to pipe system and fluid contained in it in an unsteady. The excitation process may be caused by sudden change in flow and pressure or by some mechanical action namely sudden closure of valve. This interaction result in vibration of pipe and abrupt increase in velocity and pressure of fluid. The resulting load is transferred to pipe supporting system.

There are three type of coupling governing fluid structure interaction in pipe system

1. Friction Coupling
2. Poisson Coupling
3. Junction coupling

Friction coupling is due to transient liquid shear stresses resisting relatively axial motion between fluid and pipe wall. These stress act at interface between fluid and pipe. The effect of this coupling as compared to other two is less significant. Poisson coupling is associated to hoop or circumferential stress deflection produced by liquid pressure transient which is transferred to axial stress in the ration of Poisson ratio. The axial stress and axial strain deflection traverse along pipe with speed of sound approximately. Junction coupling is most important coupling among three and its impact in most significant. It results because of reaction set up by unbalanced pressure forces and by change in fluid momentum at different position in pipe such as valve, bend etc. Sources of excitation are not concentrated only on liquid part but also they are equally contributed from solid side as in case of valve motion the change in structure moment results in liquid transient and cause vibration. Fig.1 shows the bi-directional effect of these coupling on fluid and pipe system.

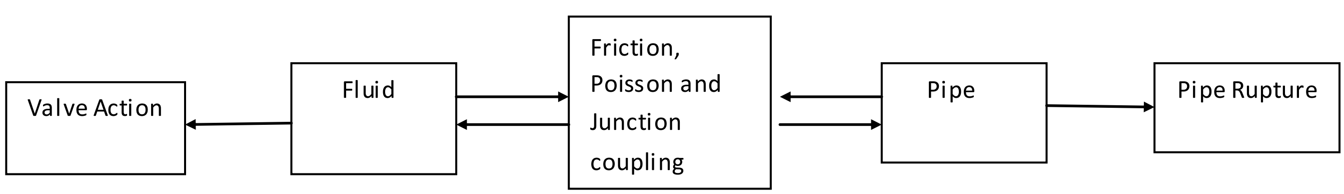

Fig.1 Symmetric diagram for interaction and excitation in liquid pipe system

There may be a deviation from practical data due to several other factors. The highly non linear nature of problem as it is coupling to physical world make the solution more tough and it get worst if we consider highly turbulent flow. The Navier-strokes equation for fluid and Euler equation for pipe has to be solved simultaneously with approximate boundary condition. In this project we have use a finite element method to solve this problem showing the effect of each coupling individually.

1.2 Literature Review

Fluid structure interaction is one of the major studies in water hammer analysis:

Tijsseling [1] carried out a very detailed review of transient phenomena in liquid-filled pipe systems. He dealt with water hammer, cavitations, structural dynamics and fluid-structure interaction (FSI). His main focus was on the history of FSI research in time domain.

Wiggert and Tijsseling [2] attempted to succinctly summarize the essential mechanisms that cause FSI, and present relevant data that describe the phenomenon. In addition, the various numerical and analytical methods that have been developed to successfully predict FSI has been described.

Heinsbroek [3] compared two different ways, MOC and finite element method for solving structural equation. Thos study showed that the FSI in pipeline system can adequately be investigated by application of MOC and FEM for hydraulic and structure of piping system, respectively. Tijsseling [4] illustrated the MOC approach for obtaining exact solution of FSI four equation model and it is generally referred in most of the latest papers for validating the results with benchmark examples.

Wiggert *et al.* [5] used a one dimensional wave formulation in both the liquid reaches and the piping structure resulting in five wave components and fourteen variables. MOC approach is introduced in this paper.

Murat [6] studied finite element analyses with scheme of Finite difference method and method of characteristics to simulate aspects of water hammer.

Le *et al.* [7] has independently solved the main FSI equation analytically. In both studies only junction coupling is considered but only for an unrestrained valve. In addition to the time domain analysis many researchers have studies the theoretical and experimental aspects in frequency domain.

Ahmadi and Keramat [8] studied the analysis of water hammer with fluid–structure interaction (FSI) to investigate junction coupling effects. Junction coupling effects were studied in various types of discrete points, such as pumps, valves and branches. The emphasis was placed on an unrestrained pump and branch in the system and the associated relations were derived for modelling them. Proposed relations were considered as boundary conditions for the numerical modelling which was implemented using the finite element method for the structural equations and the method of characteristics for the hydraulic equations.

Mahmoodi *et al.*[9] determined the position of loss of coolant accident in nuclear power plants using the transient vibration signal from a pipe rupture. A finite element formulation is implemented to include the effect of fluid-structure interaction. The coupled equations of fluid motion and pipe displacement are solved.

Kermat *et al.* [10] investigated water hammer with FSI and viscoelasticity concerns pipes made of plastic, where FSI effects are more significant than in steel pipes, simply because they are more flexible (although with thicker walls, their modulus of elasticity is lower and their

Poisson's ratio is higher). This makes FSI analysis necessary to reliably estimate the ultimate pipe stresses, elbow displacements and anchor forces, especially for designs with flexibly supported pipes. It was found herein that in the early moments of the transient event FSI is significant.

Olson and Bathe [11] have formulated finite element procedure by establishing direct symmetric model based on fluid velocity potential. Sandberg [12] has proposed a symmetric finite element model for coupled acoustic vibration between fluid and structure and impact of eigen value shifting in complex system. Lee and Park [13] presented a methodology to transform coupled pipe dynamics equation into linear form about steady state value of fluid pressure and velocity. Spectral element model is used and compared with finite element model. Xiaodong [14] in his book described the fundamental procedure for solving FSI effect by use of virtual work method and establishing formulation of displacement-Velocity potential and displacement- velocity potential- Pressure method. Elghariani [15] in his thesis has discussed formulation of equations considering effect of FSI and solving it with MOC with friction coupling as a major source of investigation. Other works [16-17] in literature mentioned the finite element formulation for the fully coupled dynamic equations of motion to include the effect of fluid–structure interaction (FSI) and applied to a pipeline system used in nuclear reactors.

1.3 Objective and scope of work

The main objective of the project is show the impact of each coupling individually in water hammer analysis for single phase fluid in simple reservoir-pipeline and valve system. Two finite element formulations are used to predict fluid structure interaction effect in water hammer. The results of two formulations have been analyzed to check the significant advantage of one over the other.

Chapter-2

MATHEMATICAL MODELLING

This chapter deals with mathematical modeling of physics present in Water hammer analysis. The formulation will give details of classical water hammer and change in equation if effect of FSI is considered.

2.1 Classical Water Hammer Theory

We consider following assumption for the study of classical water hammer theory.

1. One dimensional model is considered with average cross sectional velocity and pressure.
2. Friction is considered to be constant in our study.
3. The pipe is assumed to be fully filled in due course of study and any possibility of column separation or cavity formation is avoided.
4. There is no fluid other than water in pipe.
5. The density and other structural property as well as thermodynamic property remain constant.
6. The pipe is assumed to be straight, thin walled, linear elastic and of circular cross- section containing a weakly compressible fluid.
7. Additional effect of damping, friction and gravity are ignored in our study.

As inertia forces are neglected in radial direction for both liquid and pipe wall therefore hoop stress is linearly related to pressure:

$$\sigma_{\emptyset} = \frac{R}{e} p \tag{1}$$

where $\sigma_{\emptyset}$ -Hoop Stress, p- pressure, R- radius of pipe and e- thickness of pipe.

Classical water hammer includes only equation of momentum and equation of continuity:

$$p + \dot{V}p' + \rho_f a_f^2 V' = 0 \tag{2}$$

$$p' + \rho_f \dot{V} + {}^{f\rho_f V^2}/_{2D} = 0 \tag{3}$$

$$a^* = \sqrt{\frac{{}^{K_f}/_{\rho_f}}{\left(1 + {}^{K_f D}/_{Ee}\right)}} \tag{4}$$

where above equation representing propagation of elastic wave through fluid in pipe. a^*-Wave Speed.

2.2 Modification for classical water hammer equation

This section describes the formulation of pipe dynamics equation represent the water hammer phenomenon.

2.2.1 Governing equation For Fluid

Equation for conservation of momentum and conservation of mass for fluid considering effect of FSI as per Wiggert *et al.* [2] are:

$$\rho_f \dot{V} + P' = 0 \tag{5}$$

$$\dot{P} + \rho_f a^{*2} V' - 2\rho_f a^{*2} \nu \dot{u}' = 0 \tag{6}$$

$$a^{*2} = \frac{K_f/\rho_f}{(1-\nu^2)*(1+K_f D)/Ee} \tag{7}$$

The friction effect is not considered in our study as its role in water hammer effect is very negligible as compared to other coupling. If equation 5 and 6 are differentiated with respect to axial distance and time respectively two wave equation are observed either in terms of pressure or velocity. These are elliptical in nature and are among best suited for FEM method. As valve closure is a function of velocity so equation of wave will be represented in terms of velocity:

$$V'' - \ddot{V}/a^{*2} - 2\nu\dot{u}'' = 0 \tag{8}$$

2.2.2 Governing equation for Structural part:

The axial motion of pipe is described again in pipe dynamics equation expressed by Wiggert [2]

$$EA_s u'' - M_s u^{\cdot\cdot} + 2\vartheta A_s p' += 0 \tag{9}$$

This is also wave equation elliptical in nature without consideration of friction and pressure head due to gravity. The 2D beam element with 2 degree of freedom at each node is modeled using pipe dynamics equation.

2.3Inital and Boundary condition:

Appropriate initial and boundary condition are given to solve the benchmark problem of valve closure.

2.3.1 Initial condition

1. V(x, t=0, 0) = V_o

2. u (x, t= 0, 0) = 0

3. P(x, t= 0, 0) = $\rho_f g h_0$ ($h_0 = 0$ $is\ as\ reference\ line\ is\ pipe\ axis$)

There first and second derivative of above variable is taken as 0.

2.3.2 Boundary condition

For boundary condition as we have to simulate Poisson and Junction coupling some equation expressing physical process in boundary phenomenon should be mentioned. As junction coupling is basically interaction between fluid and solid as in case of valve we have to express valve phenomenon with some equation. For a simple reservoir-pipe-valve system with structural velocity zero and pressure head constant the boundary condition for valve is

$$\frac{\Delta P}{\rho} = \frac{CV^2}{2};\ \ \Delta F = \Delta p A_c = \frac{\rho C v V^2 A_c}{2} \tag{10}$$

$$C = \frac{C_{caliberation}}{-1.66*10^{-3}+0.1577\theta-0.535\,\theta^2+1.689\,\theta^3-0.694\,\theta^4} \tag{11}$$

$$C_{calibration}=1.475$$

2.4 Numerical Solution

In this part important issues regarding problem formulation according to solution method will be discussed.

Two methodologies have been discussed in subsequent section for analyzing water hammer effect considering effect of FSI is u-ϕ and u-ϕ-P formulation. It is known from basic knowledge of FEM that this method is not good for solving fluid equation as it is incapable of travelling discontinuities but in our case it is of less important as in our reservoir-pipe and valve system chances of random change in pressure and velocity are negligible. The general equation stating finite element method which considers damping, stiffness and mass element is

$$M\ddot{X} + C\dot{X} + KX = 0 \quad (12)$$

M- Mass Matrix, C- Damping Matrix, K- Stiffness Matrix.

2.4.1 Finite element method

A number of Finite element formulations have been proposed to model fluid for FSI analysis. Among those formulation Displacement –Velocity potential and Displacement- Velocity Potential-Pressure formulation are major breakthrough and are presented here. The comparison of results between them has been done in order to understand the difference between pros and cons of the formulation. These are the linear model and ignore material as well as geometric non linear ties. FSI interface motions are considered to be insignificant.

2.4.2 Displacement and Velocity potential formulation

Based on Hamilton principle virtual work statement of problem is mentioned below:

$$\int_0^t (\delta T - \delta U + \delta W)\,\partial t = 0 \quad (12)$$

$$T = \frac{M_P}{2}\int_0^L u^{\cdot 2}\partial x \quad (13)$$

$$U = EA_s\int_0^L (u' + \frac{1}{2}u'^2)\partial x \quad (14)$$

The total virtual force acting on structure due to flow of fluid and the resultant forces is given by

$$\delta W = \int_0^L \tau S\delta u + T_1\delta u(0) + T_2\delta u(L) \quad (15)$$

Where over dot means time derivative and prime means spatial derivative. E signifies to young modulus, L is length of pipe, A_s area of pipe and T1 and T2 are the axial forces acting on pipe at L=0 and L=L.

For one dimensional solid part the principle the Variational Indicator in defined as

$$\int[\frac{1}{2}\int \varepsilon^T\, C_s\, \varepsilon\partial x - \frac{1}{2}\int \rho_s u^T \dot{u}\, \partial x - \int u^{fsi} f^{sf}\, \partial I - \int (u^{fs})^T f^{sp}\, \partial I - \int u^T\, f^s \partial x]\partial t = 0 \qquad (16)$$

Where ε = strain tensor. Cs = material stress strain tensor, ρ_s=density, u= displacement vector, f^{sf}= Surface traction vector, f^{sp}= FSI interface traction vector, f^s= Body force vector, I = FSI region.

For compressible, inviscid fluid which experiences only small displacement and $v = \nabla\emptyset$. One dimensional fluid flow equation is:

$$\int\left[\frac{1}{2}\int \frac{1}{k}\left(\rho_f \dot{\emptyset}\right)^2 \partial x - \frac{1}{2}\rho_f (\nabla\emptyset)^2 \partial x + \int \rho_f \dot{\emptyset}^{fsi}\left(u^{fsi}\right)^T n\partial I\right]\partial t = 0 \qquad (17)$$

$\emptyset$= velocity potential, k=fluid bulk modulus; ρ_f= fluid density, n = unit outward vector

Kinematic and dynamic matching conditions along the FSI interface for linking solid and fluid part:

$$f^{sf} = \rho_f * \dot{\emptyset}^{fsi} n \qquad (18)$$

$$\dot{u}^{fsi}.n = \frac{\partial\emptyset}{\partial n} \qquad (19)$$

As per Hamilton principle all variation vanishes at t1 and t2. We solve equation15 and 16 by putting 17 and 18

$$\int \rho_s \delta\, u^T \ddot{u}\partial x + \int \delta\varepsilon^T\, C_s \varepsilon\partial x - \int (\delta u^{s,f})^T f^{s,p}\partial I - \int \delta u^T\, f^s\, \partial x - \int\left(\delta u^{fsi}\right)^T \rho_f \dot{\emptyset}^{fsi} n\partial I = 0 \quad (20)$$

$$-\int \rho_f\, (\nabla\delta\emptyset).(\nabla\emptyset)\partial x - \int \frac{\rho_f^2}{kf}\ddot{\emptyset}\delta\emptyset\partial x - \int \rho_f \dot{u}^{fsi}.n\delta\emptyset^{fsi}\partial I = 0 \qquad (21)$$

Above two matrix are expressed as discretized form and in the form of matrix:

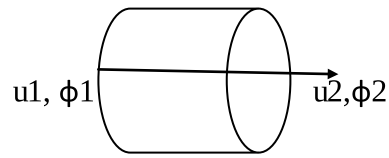

Fig.2.4.2.1 Pipe element with two degree of freedom at each node

$$\begin{bmatrix} M_{ss} & 0 \\ 0 & -M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{\emptyset} \end{Bmatrix} + \begin{bmatrix} C_{fs} & 0 \\ 0 & C_{fs}^{T} \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{\emptyset} \end{Bmatrix} + \begin{bmatrix} K_{ss} & 0 \\ 0 & -K_{ff} \end{bmatrix} \begin{Bmatrix} U \\ \emptyset \end{Bmatrix} = \begin{Bmatrix} F_{s1} \\ F_{f1} \end{Bmatrix} \qquad (22)$$

Solution for displacement –Velocity potential formulation:

[d]- displacement field, [F]= force vector,[di]and [do]=initial condition ,

$$[d]=\begin{bmatrix} u1 \\ u2 \\ \emptyset 1 \\ \emptyset 2 \end{bmatrix},\ [F]=\begin{bmatrix} \{Fs1\}_{2*1} \\ \{Ff1\}_{2*1} \end{bmatrix},\ [di]=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix},\ [do]=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (23)$$

2.4.3 Displacement, velocity Potential and pressure formulation

In this kind of formulation we need one hydrostatic pressure unknown P to replace one nodal velocity potential in last section. The discretized form of equation obtained from the similar procedure followed in above section in form of matrix is

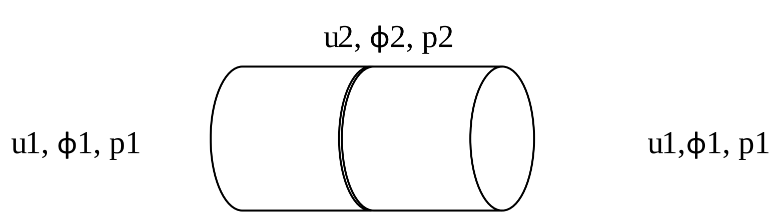

Fig2.4.3 Pipe element with 3 degree of freedom at three nodes

$$\begin{bmatrix} M_{ss} & 0 & 0 \\ M_{sf} & -M_{ff} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U^{\cdot\cdot} \\ \emptyset^{\cdot\cdot} \\ p^{\cdot\cdot} \end{Bmatrix} + \begin{bmatrix} 0 & C_{fs}^T & 0 \\ C_{fs} & 0 & C_{pf}^T \\ 0 & C_{pf} & 0 \end{bmatrix} \begin{Bmatrix} U^{\cdot} \\ \emptyset^{\cdot} \\ p^{\cdot} \end{Bmatrix} + \begin{bmatrix} K_{ss} & 0 & K_{ps}^T \\ 0 & -K_{ff} & 0 \\ K_{ps} & 0 & K_{pp} \end{bmatrix} \begin{Bmatrix} U \\ \emptyset \\ p \end{Bmatrix} = \begin{Bmatrix} F_{s2} \\ F_{f2} \\ F_v \end{Bmatrix} \qquad (24)$$

Solution for displacement velocity potential and pressure formulation:

[d]- Displacement field, [F]= force vector,[di]and [do]=initial condition

$$[d] = \begin{bmatrix} u1 \\ u2 \\ u3 \\ \phi1 \\ \phi2 \\ \phi3 \\ P1 \\ P2 \\ P3 \end{bmatrix}, \quad [di] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, [do] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, [F] = \begin{bmatrix} \{Fs\}_{3*1} \\ \{Ff\}_{3*1} \\ \{Fv\}_{3*1} \end{bmatrix} \qquad (25)$$

The above two formulation have been used for study of fluid structure interaction. We have assumed an in viscid, ir-rotational compressible fluid with small motion in contact with elastic solids (pipe). In $\emptyset - U$, we use $\emptyset$ velocity potential as state variable for fluid and displacement for solid. In second type of formulation we have introduce pressure because this method offers symmetry formulation with reduced fluid degree of freedom, it can be easy incorporated in displacement based program. The coupling node in structural and fluid domain has same global co-ordinates; hence interface element has zero mass.

The mass, stiffness and damping matrix has been established with force matrix representing basic finite element formulation. In next section we will calculate the matrix element.

2.4.4 Interpolation or Shape function

As we are solving two case when pipe has two nodes with 2 degree of freedom at each node in first formulation discussed in 2.4.2 and in 2.4.3 we consider a pipe with three node with 3 degree of freedom at each node. The finite element model is modeled by using displacement and fluid field.

$$u(x,t) = [N_u(x)][d_u(t)] \quad (26)$$

$$\Phi(x,t) = [N_f(x)][d_f(t)]$$

$$P(x,t) = [N_p(x)][d_p(t)]$$

Where $[N_u(x)], [N_p(x)]\ and [N_f(x)]$ represent the shape function for structural and fluid part respectively. $[d_u(t)]$, $[d_f(t)]$ and $[d_p(t)]$ are the displacement vectors. The selection of shape function depends upon degree of freedom. Isoparametric function are used for all degree of freedom related to fluid, structure and interface. In u-Φ formulation we have 2 degree of freedom per node therefore we choose a linear shape function and this shape function can be used for structure and fluid for making calculation easy.

$$[N_u(x)] = [N_p(x)] = [N_f(x)] = [1 - \tfrac{x}{L} \quad \tfrac{x}{L}] \qquad 0 \le x \le L \qquad (27)$$

$$d_u(t) = [u1 \quad u2]^T \text{ and } d_f(t) = [\Phi 1 \quad \Phi 2]^T$$

For second case of u-Φ-P formulation as degree of freedom per node are 3. To approximate this behavior with fluid finite element we have used quadratic shape function as pressure field is a solution to Helmholtz equation which is characterized by sinusoidal variation in space.

$$[N_u(x)] = [N_p(x)] = [N_f(x)] = [-(1-2x)/2 \quad (1+2x)/2 \quad -2x] \qquad 0 \le x \le L \quad (28)$$

$$d_u(t) = [u1 \quad u2 \quad u3]^T, d_f(t) = [\emptyset 1 \quad \emptyset 2 \quad \emptyset 3]^T and\ d_P(t) = [P1 \quad P2 \quad P3]^T$$

2.4.5 Matrix element formulation in terms of shape function

Using these shape function and expressing for mass, stiffness and coupling matrix have been derived from the above discussion in 2.4.2 and 2.4.3. They are defined as

1. Mass Matrix Element:

$$M_{ss} = M_s \int_0^l [N_s]^T [N_s] \partial x,\ M_{ff} = \frac{\rho_f^2}{K_f} \int_0^l [N_f]^T [N_f] \partial x \text{ and } M_{fs} = \frac{c_0}{a^2} \int_0^l [N_f']^T [N_s'] \partial x \tag{29}$$

2. Stiffness matrix element

$$K_{ff} = \rho_f \int_0^l [N_s']^T [N_s'] \partial x ,K_{sf} = \frac{-Mwfc_0}{D} \int_0^l [N_s]^T [N_f] \partial x,\ K_{ss} = EA_s \int_0^l [N_s']^T [N_s'] \partial x, \tag{30}$$

$$K_{pp} = \frac{1}{K_f} \int_0^l [N_p]^T [N_p] \partial x \text{ and } K_{ps} = \int_0^l [N_p]^T [N_s] \partial x$$

3. Damping(coupling) matrix element

$$C_{ff} = \frac{c_0}{a^2} \left[\int_0^l [N_f\ ]^T [N_f'] \partial x + \frac{f}{D} \int_0^l [N_f]^T [N_f] \right] \partial x, \quad C_{fs} = -\rho_f \vartheta \int_0^l [N_f']^T [N_s'] \partial x \quad \text{and} \tag{31}$$

$$C_{pf} = \frac{\rho_f}{K} \int_0^l [N_p]^T [N_f] \partial x$$

4. (a) Force Matrix for u-Φ formulation

$$F_{s1} = \frac{Mwfc_0{}^2}{K} \int_0^l [N_f]^T \partial x + \int_0^l \frac{\rho C_v([N_p]^T x)^2 A_c}{2} \partial x \text{ and } F_{f1} = \int_0^l [N_s]^T \partial x + \int_0^l \frac{\rho (C_v[N_p]^T x)^2 A_c}{2} \partial x \tag{32}$$

(b) Force matrix for u-Φ-P formulation

$$F_{s2} = \frac{Mwfc_0{}^2}{K} \int_0^l [N_f]^T \partial x,\quad F_{f2} = \int_0^l [N_s]^T \partial x \text{ and } Fv = \int_0^d \frac{\rho C_v([N_p]^T x)^2 A_c}{2} \partial x \tag{33}$$

2.4.6 Solution Procedure

As the simulation of water hammer including FSI effect is modeled for one dimension mathematical model we choose the numerical values of various parameters used by

Serial no.	Parameter	Numerical Value
1	Length(m)	20
2	Outer diameter(m)	0.813
3	Thickness(m)	0.008
4	Young's Modulus	$2.1*10^{11}$ N/m^2
5	Bulk Modulus	$2.1*10^{9}$ N/m^2
6	Poisson's Ratio	0.3
7	Gravitational acc.(m/sec$^{2)}$	9.8
8	Fluid velocity(m/sec)	1
9	Pressure behind valve(N/m^2)	0
10	Density of pipe(Kg/m^3)	7900
11	Density of fluid(Kg/m^3)	1000
12	Wave velocity(a*)(m/sec)	1020

Table2.1 Specification of piping system

A MATLAB program was written following New mark scheme with alpha=0.5 and gamma=0.1677 to solve for all degree of freedom u, p and Φ and obtain their variation with time. To examine the validation of proposed model the results were compared with Ahmadi and Keramat [8].There is no friction and valve closure time in 4 sec.

Chapter-3

RESULT AND DISCUSSION

3.1 Result and analysis for Displacement, Velocity Potential and Pressure Formulation

The variation of various parameters like displacement amplitude, velocity, pressure and forces with time has been discussed

3.1.1 Displacement variation with time

Fig.3.1.1 shows the variation of displacement at different nodes with time. It can be observed from the graph that at node 3 which is actually place for junction coupling suffers maximum displacement accompanied by fluctuation as valve is closed gradually with time. We can also observe that as we move from valve end to reservoir end (from node 3 to node 1) the displacement amplitude decreases. The results are in good accordance with experimental result.[8]

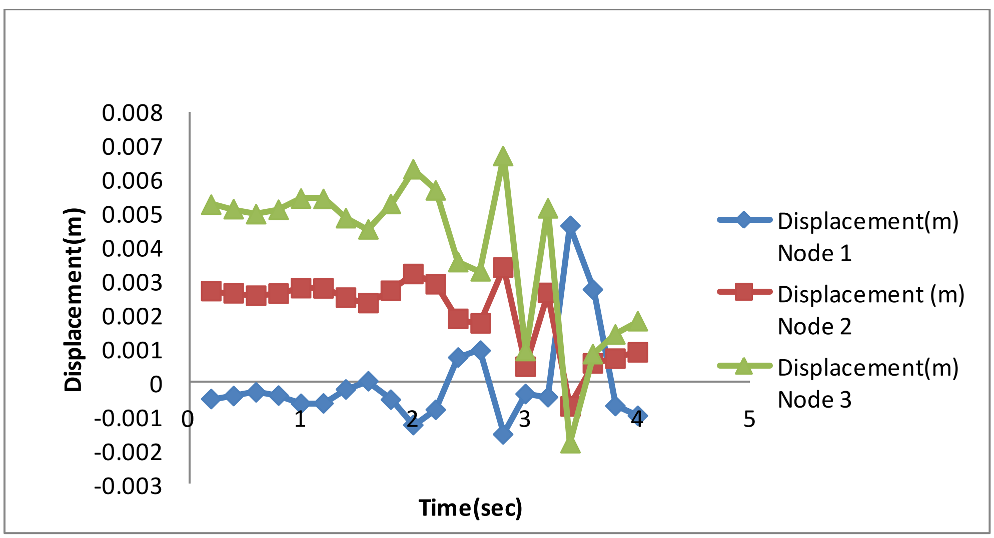

Fig.3.1.1: Displacement (m) Vs Time (sec) for u-ϕ-P method for 3 nodes

3.1.2 Velocity variation with Time

Discussion

The variation of velocity with time is very well expressed from Fig.3.1.2 as it is in great similarity with experimental result. The variation of velocity with length of pipe (different node) suggests that as the valve will be closed the velocity will increase randomly from valve end to reservoir end. The quantitative value gives an idea of large hydraulic force acting on structure.

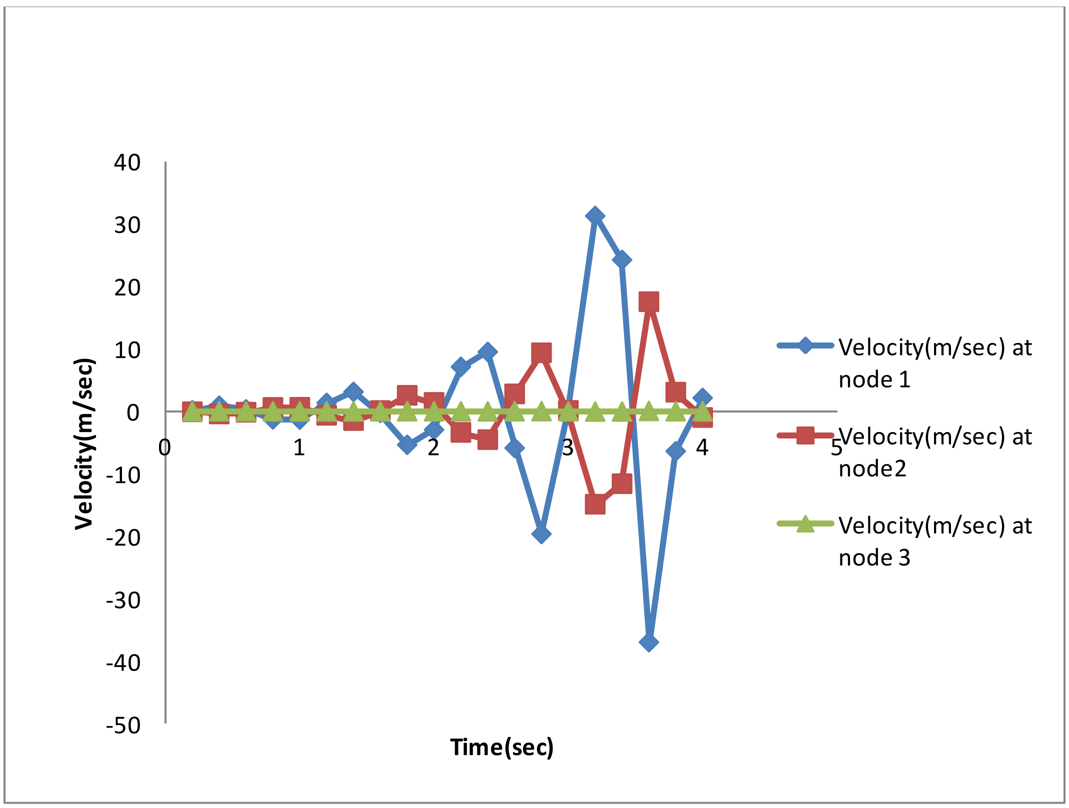

Fig .3.1.2 Velocity (m/s) Vs Time (sec) for u-ϕ-P method for 3 nodes

3.1.3Pressure variation with time

Fig.3.1.3 shows the comparison between pressure amplitude between classical water hammer solution and FEM approach. There is been deviation in pressure amplitude measured from both method. The deviation is because of FSI effect considered in our approach. Classical water hammer result gives pressure amplitude of 300000 N/m^2 where as our simulation result predicts peak pressure of 1000000 N/m^2. This result is corresponding to FSI effect caused by various coupling.

$$\Delta p_{classical} = \rho_f a^* V \qquad (34)$$

It also shows the random increase in pressure amplitude which is attributed to junction coupling effect, as the valve is closed gradually the pressure increases and velocity decreases(from previous result). It is also suggested from above graph that pressure amplitude falls with the length of pipe from valve to reservoir end. Peaks in the graph suggest the possible reason for structure deformation as the change in pipe amplitude is not gradual. The experimental results are in great accordance with it.

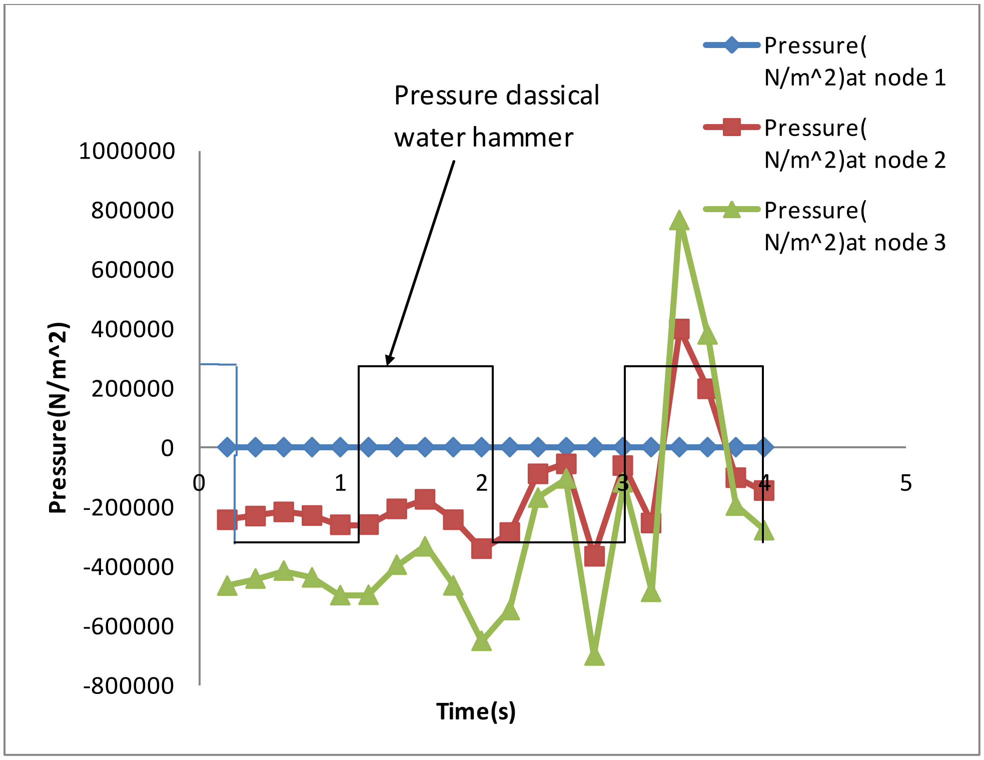

Fig.3.1.3 Pressure (N/m^2) Vs Time (sec) for u-ϕ-p method for 3 nodes

3.1.4 Force on Solid (pipe) variation with time at each pipe node:

Fig.3.1.4 presents a quantitative idea of force acting on structure (pipe) across the length of pipe. The force is maximum at node 3 which is accordance of theory because of valve closure there will be sudden rise in pressure and therefore force acting on valve due to fluid increases and as the pressure decreases along the length from valve to reservoir the force amplitude decrease. The values for force at different node are helpful in deciding the material of pipe and valve for a particular physical solution and are very important for design consideration.

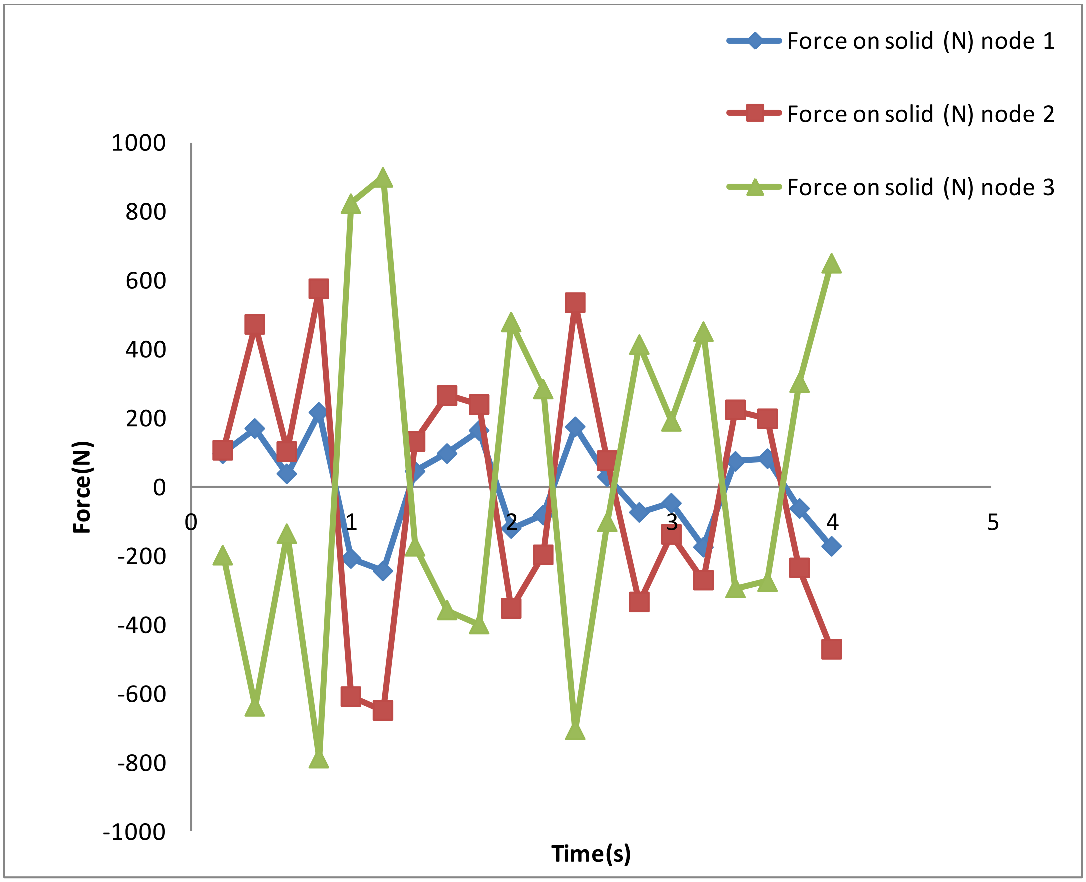

Fig 3.1.4 Force on structure (N) Vs Time (sec) for u-ϕ-P method for 3 nodes

3.1.5 Force on fluid vs time variation at all nodes

Fig.3.1.5 states that the force acting on fluid due to structure is very less as compared to fluid on structure this is because of highly rigid nature for pipe. For elastic pipes this force will be almost of same order of Force on structure (Pipe). This force gradually increases wave velocity of fluid and this gradual increase in fluid velocity results in gradual increase of pipe displacement.

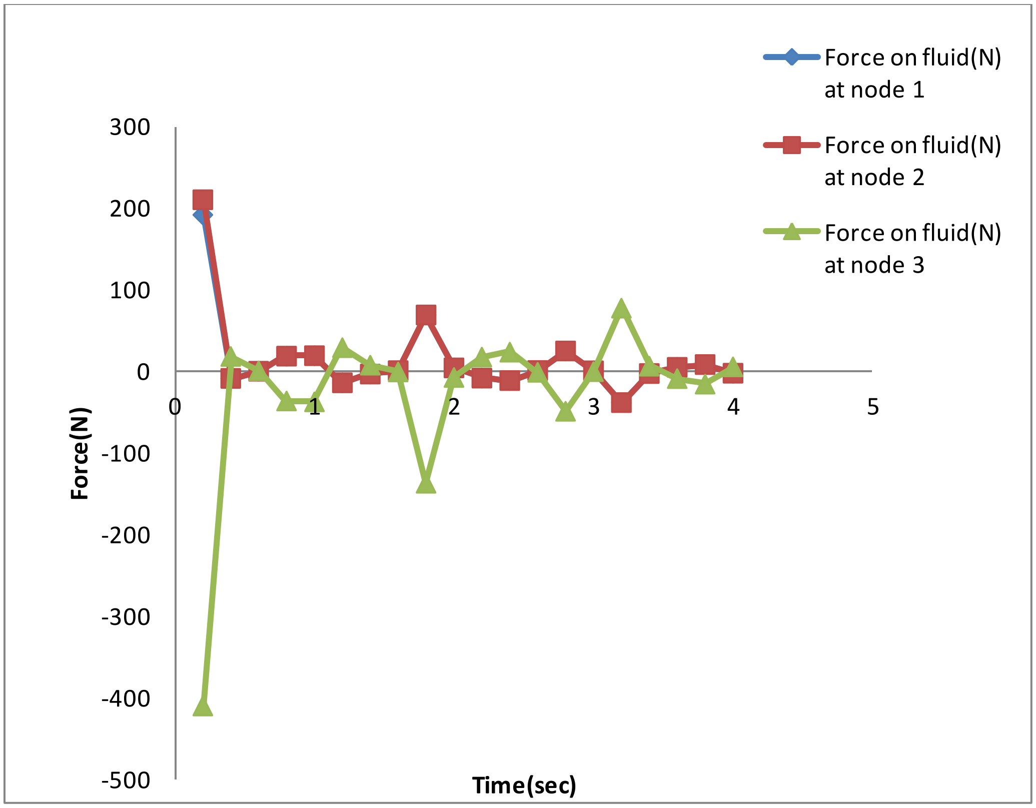

Fig 3.1.5 Force on fluid (N) Vs Time (sec) for u-ϕ-P method for 3 nodes

3.2 Result and analysis for Displacement and Velocity Potential formulation:

Fig.3.2.1-3.2.4 Time history for displacement amplitude, velocity and forces are obtained after solving nonlinear equation; it is found in great similarity with published result [8]. The discussion has already been made in previous section. It is important to consider that forces exerted on solid and fluid are same as compared to previous result, the reason for this it is probably the nature of formulation where we added hydraulic force and force acting on valve to both force acting on structure due to fluid and vice versa.

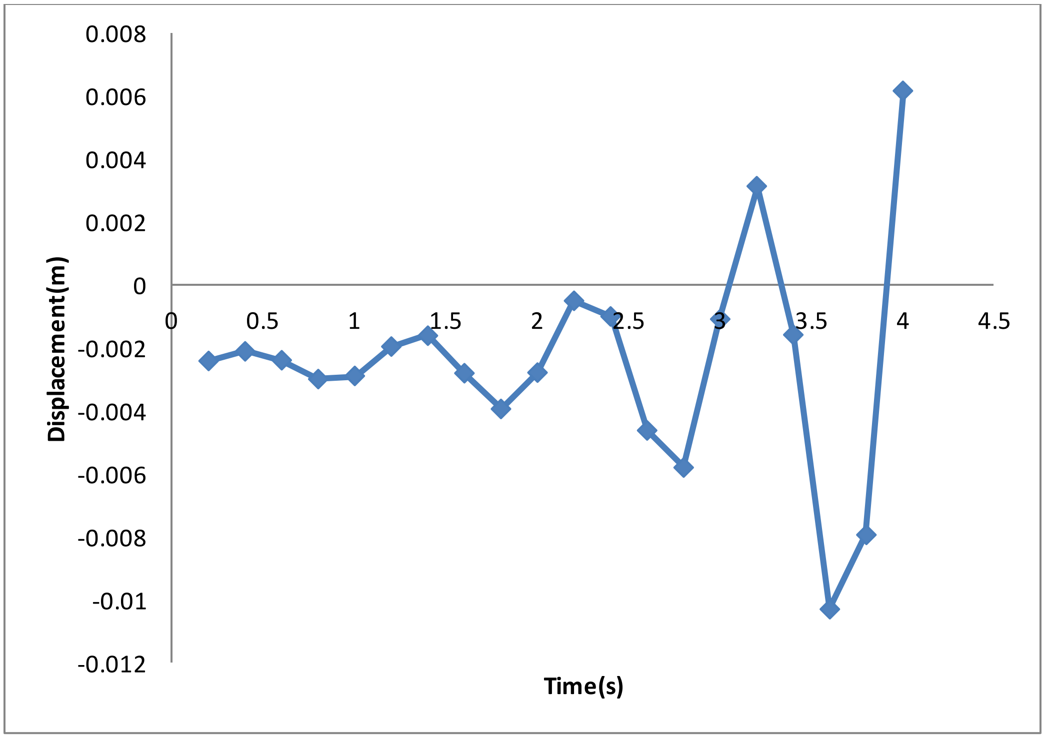

Fig.3.2.1 Displacement (m) Vs Time (sec) for u-ϕ method

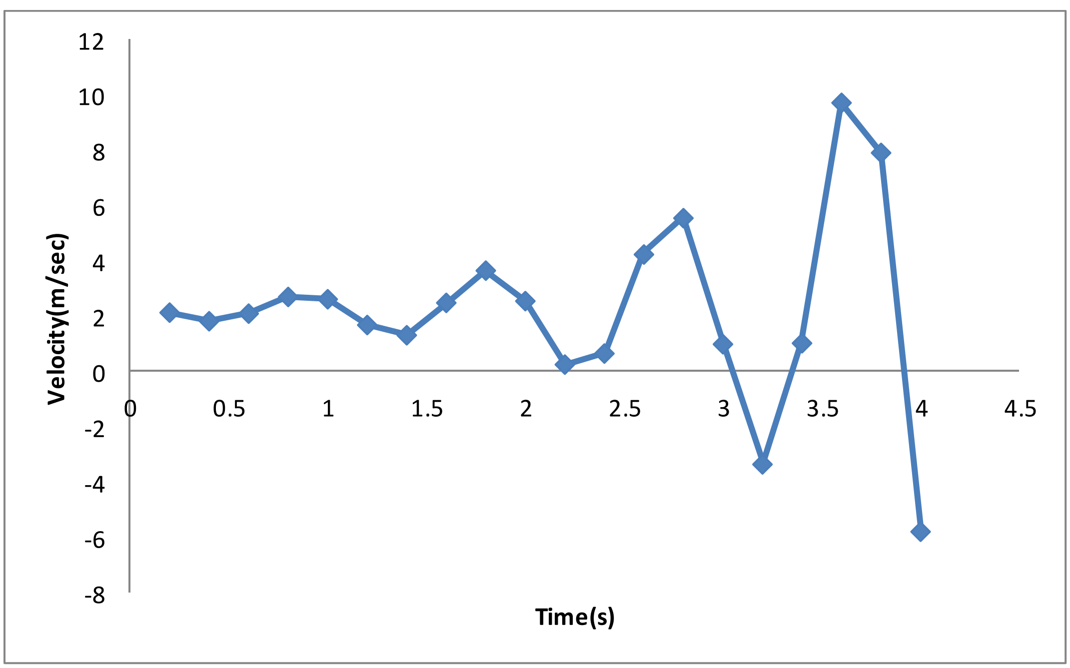

Fig.3.2.2 Velocity (m/sec) Vs Time (sec) for u-ϕ method

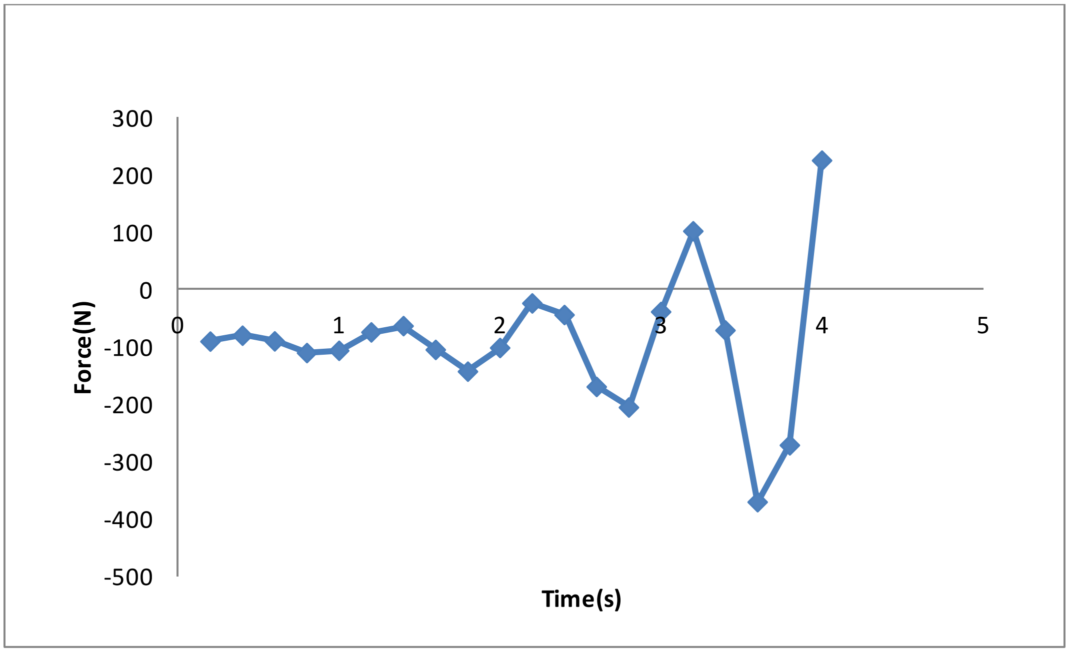

Fig.3.2.3 Force on structure (N) Vs Time (sec) for u-ϕ method

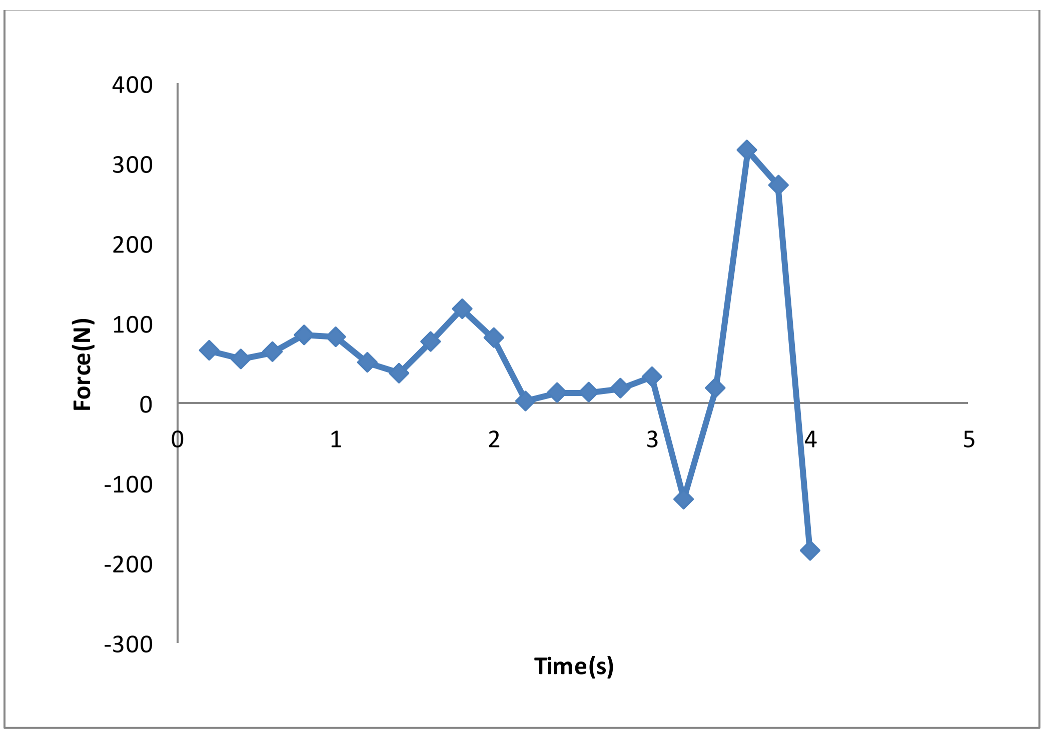

Fig.3.2.4: Force on fluid (N) Vs Time (sec) for u-ϕ method

3.3 Comparison of individual effect of coupling

The comparison is done by taking account of three major cases

1. Considering effect of Poisson coupling
2. Considering effect of Junction coupling
3. Considering effect of both Junction and Poisson coupling

Fig.3.3.1-3.3.4 presents a very important prospect of the result as we found that displacement, velocity and force variation is very less as compared to case where both the coupling are considered for formulation. This suggest that in case pipe material has high rigidity value the Poisson coupling is of less importance as its contribution to water hammer is small. If the pipe material is elastic in nature then the effect of Poisson coupling will surely increase and therefore become an important consideration for calculating the water hammer effect in elastic pipes.

From Fig.3.3.1-3.3.4 It is found that when we consider only junction coupling without considering the impact of other two coupling the displacement, velocity and force variation do not differ a lot from the case where all coupling have been considered. This suggest that in case of valve closure boundary phenomenon with rigid pipe junction coupling play an important role in water hammer and is reason for pipe and valve rupture. It is also observed that pipe undergo compression at start of valve closure thereby by putting a huge pressure on liquid.

It is also evident that both coupling when acted together produced enhanced amplitude of all variables displacement, velocity and forces than individual coupling. This is because when Poisson coupling come into picture it directly relates the pressure with stress of pipe and fluid velocity with change in pipe displacement, therefore both domain(fluid and solid) response to change in each other dynamically which is not possible in case of only junction coupling alone. But our factor also depends upon on rigidity modulus of pipe material if it is high the role of Poisson coupling will be small and if its is small then its role be large. In our case Poisson coupling contributes to small deformation or change in Displacement amplitude.

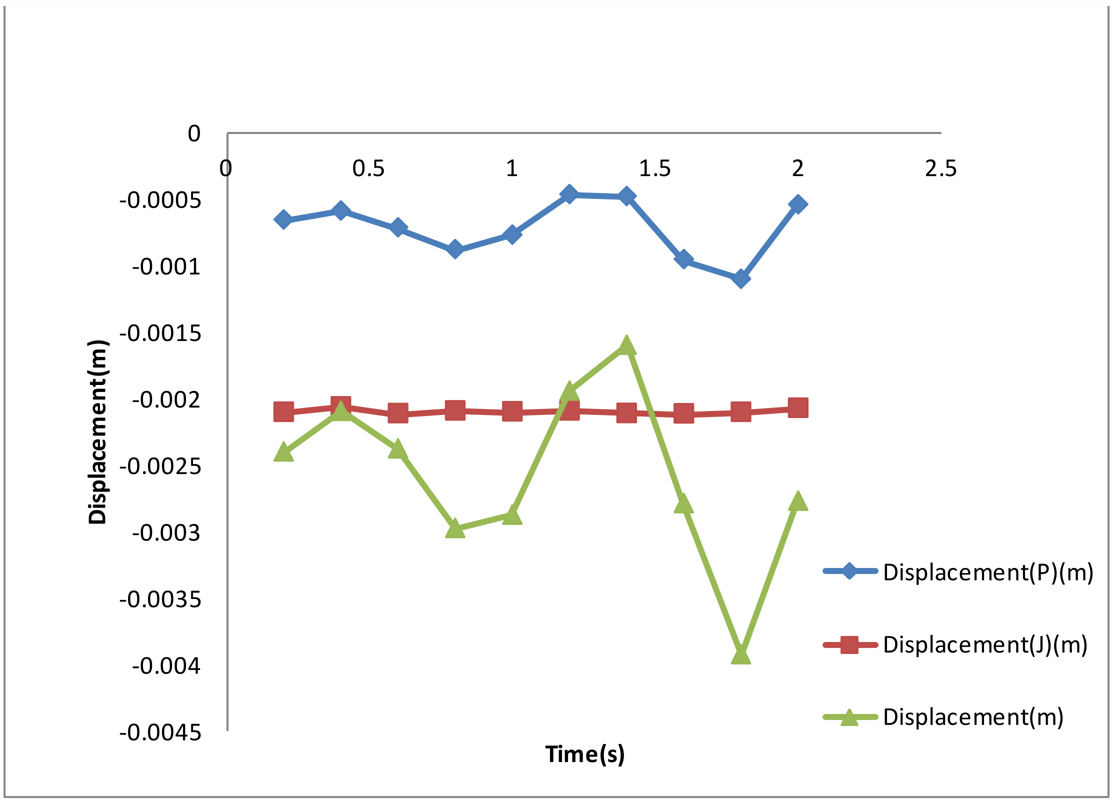

Fig.3.3.1 Variation of Displacement (m) Vs Time (sec) for u-ϕ method with consideration of all coupling

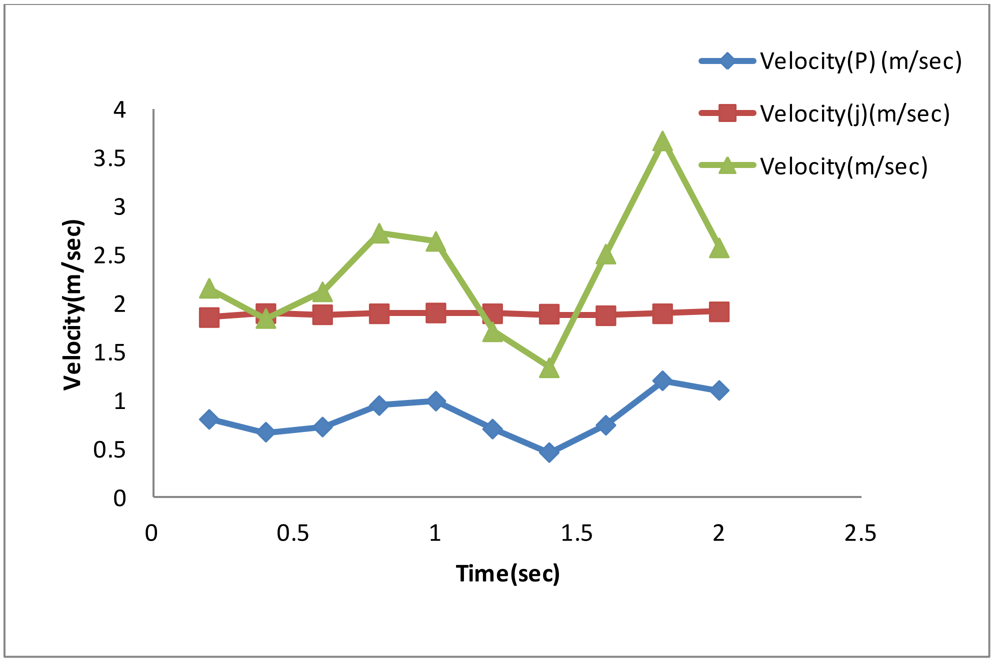

Fig 3.3.2: Variation of Velocity (m/s) Vs Time (sec) for u-ϕ method with consideration of all coupling

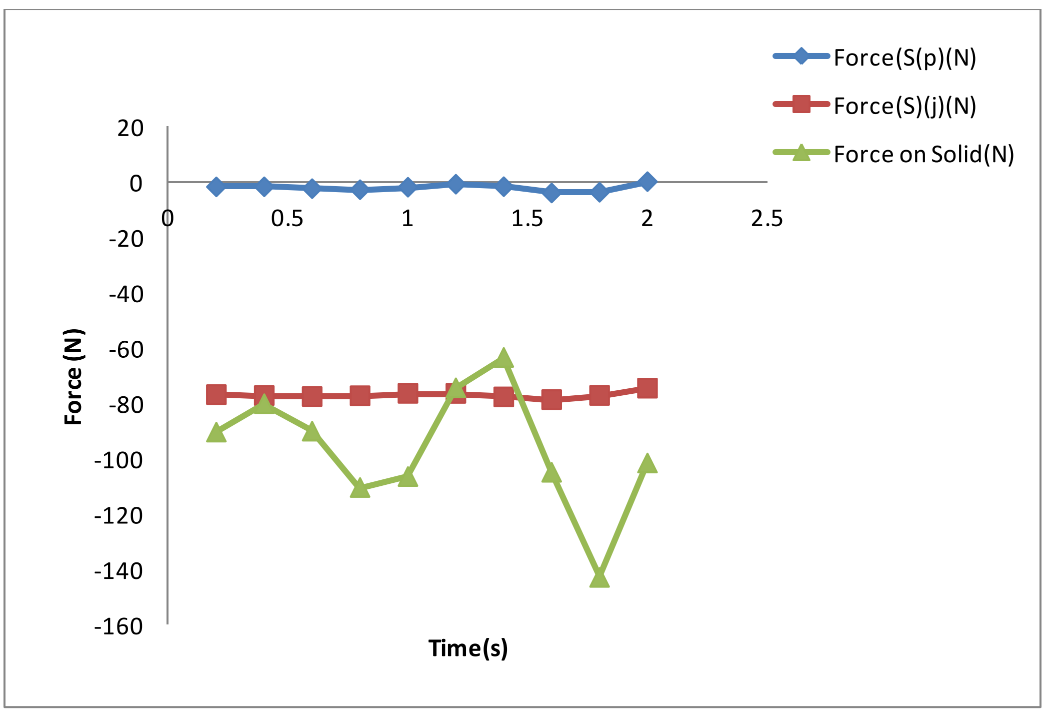

Fig.3.3.3 Variation of Force on structure (m) Vs Time (sec) for u-ϕ method with consideration of all coupling

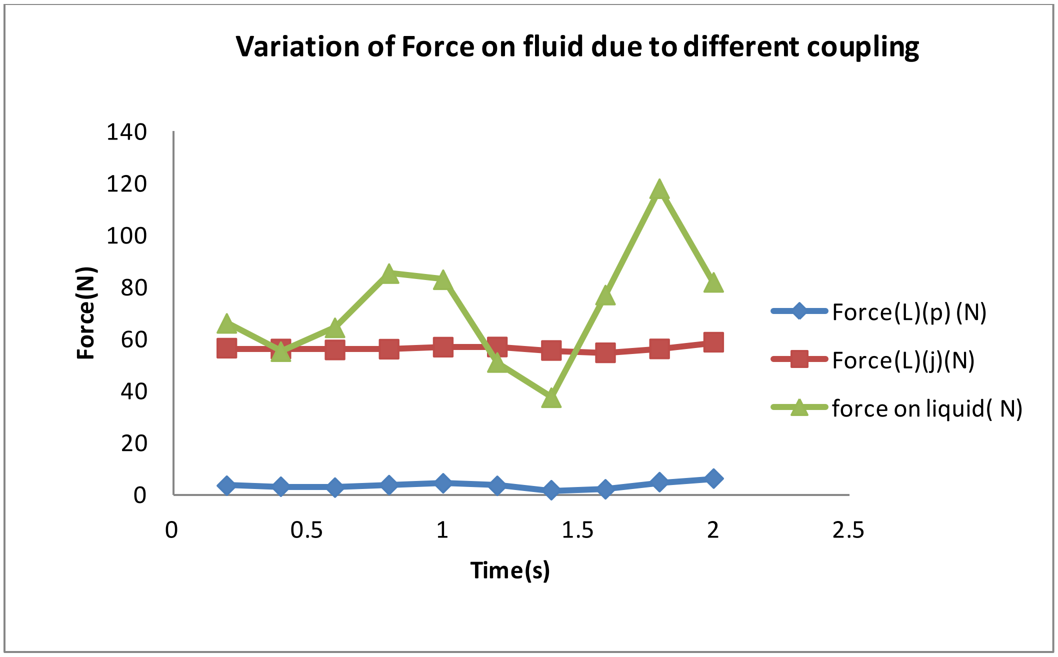

Fig.3.3.4 Variation of Force on fluid (N) Vs Time (sec) for u-ϕ method with consideration of all coupling

3.4 Validating use of u-ϕ-p formulation as compared to u-ϕ.

From Fig.3.4.1It is found that displacement amplitude variation with time calculated from both u-ϕ and u-ϕ-P formulation is very close to each other. This numerical result validates the theoretical prediction of Olson and Sandberg [9, 10] in which they described that both the formulation results in almost same solution. It is also evident from our study that using u-ϕ-p formulation gives more accurate result as we have introduced one more degree of freedom called pressure and calculates its variation at FSI interface.

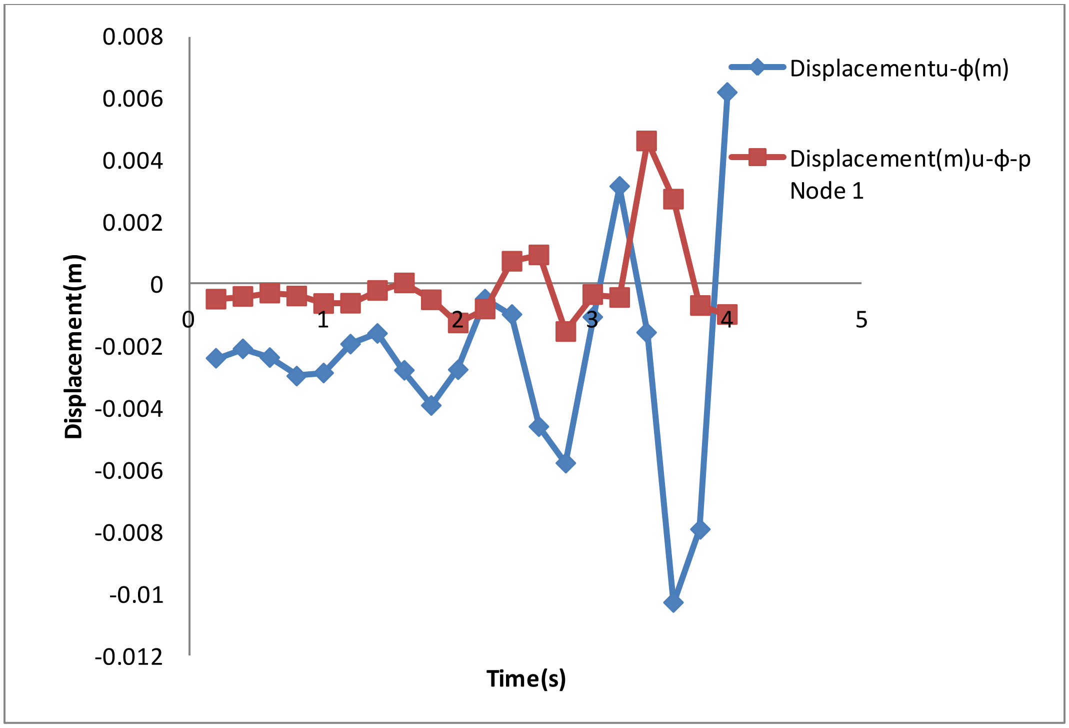

Fig .3.4.1: Comparison of Displacement (m) Vs Time (sec) for u-ϕ and u-ϕ-P method with consideration of all coupling

Chapter 4

CONCLUSIONS

4.1 Summary of Result

This report attempts to get a coupled solution for water hammer problem in straight pipe with valve closure at an end and reservoir at other end. Time histories for displacement, velocity, pressure and forces were compared along the length of pipe. We also tried to show effect of Junction and Poisson coupling as compared to classical water hammer. Some of the important conclusions drawn out of study are given below.

[1] The variation of Displacement suggests that it is maximum near valve and slowly reduces towards reservoir.

[2] The pressure peaks are found to be as large as 1000000 N/m^2, this data reveals the destructive nature of this phenomenon as the pressure has raised 10 times more than atmospheric pressure which is great concern for safe design.

[3] The force acting on structure is more than the forces acting on fluid. The values obtained from our study provide a great idea to set up designer about valve and pipe material selection so that it can bear such instant high loads.

[4] For non FSI simulation the predicted value are as much as 45% less for velocity as compared to simulation carried out by considering the effect of FSI which suggests the importance of our study and its use for safe design.

[5] The Junction coupling will play a major role in water hammer than poison coupling. In our result Junction coupling is contributing to water hammer effect by more than 70%

[6] The u-ϕ and u-ϕ-p formulation results do not vary much as suggested by OLSON, this theoretical prediction has been validated .The result also shows that u-ϕ-p formulation predicted the variation of displacement and pressure in great accordance to practical data.

[7] This kind of formulation for predicting water hammer is validated as compared to other method like MOC, mixed MOC-FEM and other FEM techniques.

4.2 Future Scope of Work

The method discussed over here provides a great deal of understanding of such difficult non linear formulation by FEM. It is advised to work with interpolation function and follow nonlinear integration scheme to predict result more accurately.

It is also suggested to take this simulation to next higher level and simulate it for two dimensions or with mix phase of fluid to give more practical touch to problem.

The variety of pipe with different stiffness and rigidity can be tested and analyzed so that suitable selection of pipe and valve can be made.

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APPENDIX

NUMERICAL VALUE FOR MATRIX and MATLAB PROGRAM

(a) Numerical values for mass, stiffness and damping matrices have been calculated and presented in matrix format

The formulation of matrix for both u-ϕ .

$$[M] = \begin{bmatrix} 6393.26 & 3196.63 & 0 & 0 \\ 3196.63 & 6393.26 & 0 & 0 \\ 0 & 0 & 3.16 * 10^{-3} & 1.58 * 10^{-3} \\ 0 & 0 & 1.58 * 10^{-3} & 3.16 * 10^{-3} \end{bmatrix}$$

$$[K] = \begin{bmatrix} 3950000000 & -3950000000 & 0 & 0 \\ -3950000000 & 3950000000 & 0 & 0 \\ 0 & 0 & 6666.66 & 3333.3 \\ 0 & 0 & 3333.33 & 6666.66 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 30 & -30 & 0 & 0 \\ -30 & 30 & 0 & 0 \\ 0 & 0 & 30 & -30 \\ 0 & 0 & -30 & 30 \end{bmatrix},$$

$$[F] = \begin{bmatrix} -3.34 - 35.76 * Cv \\ 5.31 + 25.26 * Cv \\ -2 - 35.76 * Cv \\ 3 + 25.26 * Cv \end{bmatrix}$$

The formulation for matrix for and u-ϕ-P are:

$$[M] = \begin{bmatrix} 77906.32 & 83895.52 & -161802.56 & 0 & 0 & 0 & 0 & 0 & 0 \\ 83895.52 & 90514.72 & -174410.56 & 0 & 0 & 0 & 0 & 0 & 0 \\ -161802.56 & -174410.56 & 336213.43 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.2 * 10^{-6} & 1.2 * 10^{-6} & -1.2 * 10^{-6} & -1.2 & -1.33 & 2.56 & 0 & 0 & 0 \\ 1.2 * 10^{-6} & 1.2 * 10^{-6} & -1.2 * 10^{-6} & -1.33 & -1.43 & 2.76 & 0 & 0 & 0 \\ -1.2 * 10^{-6} & -1.2 * 10^{-6} & 1.2 * 10^{-6} & 2.56 & 2.76 & -5.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, [F] = \begin{bmatrix} 94.7 \\ 104.76 \\ -199.95 \\ 190 \\ 210 \\ -400 \\ 1.404 * Cv \\ 51.63 * Cv \\ -64.10 * Cv \end{bmatrix}$$

$[K]$

$$
= \begin{bmatrix}
79000000 & 79000000 & -158000000 & 0 & 0 & 0 & 2431.66 & 2661.66 & -5133.33 \\
79000000 & 79000000 & -158000000 & 0 & 0 & 0 & 2661.66 & 2871.66 & -5533.33 \\
-158000000 & -158000000 & 316000000 & 0 & 0 & 0 & -5133.33 & -5533.33 & 10666.67 \\
0 & 0 & 0 & -2.47 \ast 10^{6} & -2.6 \ast 10^{6} & 5.13 \ast 10^{6} & 0 & 0 & 0 \\
0 & 0 & 0 & -2.6 \ast 10^{6} & -2.87 \ast 10^{6} & 5.53 \ast 10^{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 5.13 \ast 10^{6} & 5.53 \ast 10^{6} & 10.66 \ast 10^{6} & 0 & 0 & 0 \\
2431.66 & 2661.66 & -5133.33 & 0 & 0 & 0 & -1.11 \ast 10^{-3} & -1.26 \ast 10^{-3} & 2.44 \ast 10^{-3} \\
2661.66 & 2871.66 & -5533.33 & 0 & 0 & 0 & -1.26 \ast 10^{-3} & -1.36 \ast 10^{-3} & 2.63 \ast 10^{-3} \\
-5133.33 & -5533.33 & 10666.67 & 0 & 0 & 0 & 2.44 \ast 10^{-3} & 2.63 \ast 10^{-3} & -5.07 \ast 10^{-3}
\end{bmatrix}
$$

$[C]$

$$
= \begin{bmatrix}
0 & 0 & 0 & -600 & -600 & 1200 & 0 & 0 & 0 \\
0 & 0 & 0 & -600 & -600 & 1200 & 0 & 0 & 0 \\
0 & 0 & 0 & 1200 & 1200 & -2400 & 0 & 0 & 0 \\
-600 & -600 & 1200 & 0 & 0 & 0 & 90.46 \ast 10^{-6} & 90.46 \ast 10^{-6} & -180.92 \ast 10^{-6} \\
-600 & -600 & 1200 & 0 & 0 & 0 & 99.98 \ast 10^{-6} & 99.98 \ast 10^{-6} & -199.92 \ast 10^{-6} \\
1200 & 1200 & -2400 & 0 & 0 & 0 & -190.44 \ast 10^{-6} & -190.44 \ast 10^{-6} & 380.88 \ast 10^{-6} \\
0 & 0 & 0 & 90.46 \ast 10^{-6} & 90.46 \ast 10^{-6} & -180.92 \ast 10^{-6} & 0 & 0 & 0 \\
0 & 0 & 0 & 99.98 \ast 10^{-6} & 99.98 \ast 10^{-6} & -199.92 \ast 10^{-6} & 0 & 0 & 0 \\
0 & 0 & 0 & -190.44 \ast 10^{-6} & -190.44 \ast 10^{-6} & 380.88 \ast 10^{-6} & 0 & 0 & 0
\end{bmatrix}
$$

(b) MATLAB Program

Matlab Code of Integration of N*N matrix using a Newmark Scheme:

The code predict time dependent values for displacement, velocity, pressure and forces. The graph is obtained with the help of excel. This code provide flexibility to user to N*N dimensional matrix depending upon nature of problem.

alpha=0.5 and gamma =0.1677

b.1 For u-ϕ formulation (2*2 matrix)

```
function [u,ud,udd] = Water Hammer( )
format long
n=input('no. of time increments required n=');
dt=input('time interval d t =');
t(1)=0;
for i=1:n
t(i+1)=t(i)+dt;
Cv=1.78*(i*dt)^0.239;
end
```

```
disp('input the stiffness and mass matrices');
k=input('stiffness matrix=');
m=input('mass matrix=');
c=input('damping matrix c=');
disp('stiffness matrix k=');
disp(k);
disp('mass matrix m=');
disp(m);
disp('damping matrix c=');
disp(c);
u=[0;0;1;1];
ud=[0;0;0;0];
udd=[0;0;0;0];
alpha=input('integration constantl alpha =');
gamma=input('integration constant gamma=');
a0=1/(alpha*dt^2);
a1=gamma/(alpha*dt);
a2=1/(alpha*dt);
a3=(1/(2*alpha))-1;
a4=(gamma/alpha)-1;
a5=((gamma/alpha)-2)*(dt/2);
a6=dt*(1-gamma);
a7=gamma*dt;
keff=k+a0*m+a1*c;
for i=1:n
f=[-3.34-35.76*Cv; 5.31+25.26*Cv; -2-35.76*Cv; 3+25.26*Cv];
disp('force matrix f=');
disp(f);
s = m*(a0*u+a2*ud+a3*udd)+c*(a1*u+a4*ud+a5*udd);
feff=f+s;
disp('feffective ::')
disp(feff)
temp1=u;
u=inv(keff)*feff;
temp=udd;
udd=a0*(u-temp1)-a2*(ud)-a3*(udd);
ud=ud+a6*temp+a7*udd;
disp('upadated value of u:')
disp(u)
% plot(t(i)+dt,u(i));
disp('upadated value of ud:')
disp(ud)
disp('upadated value of udd:')
disp(udd)
end
end
```

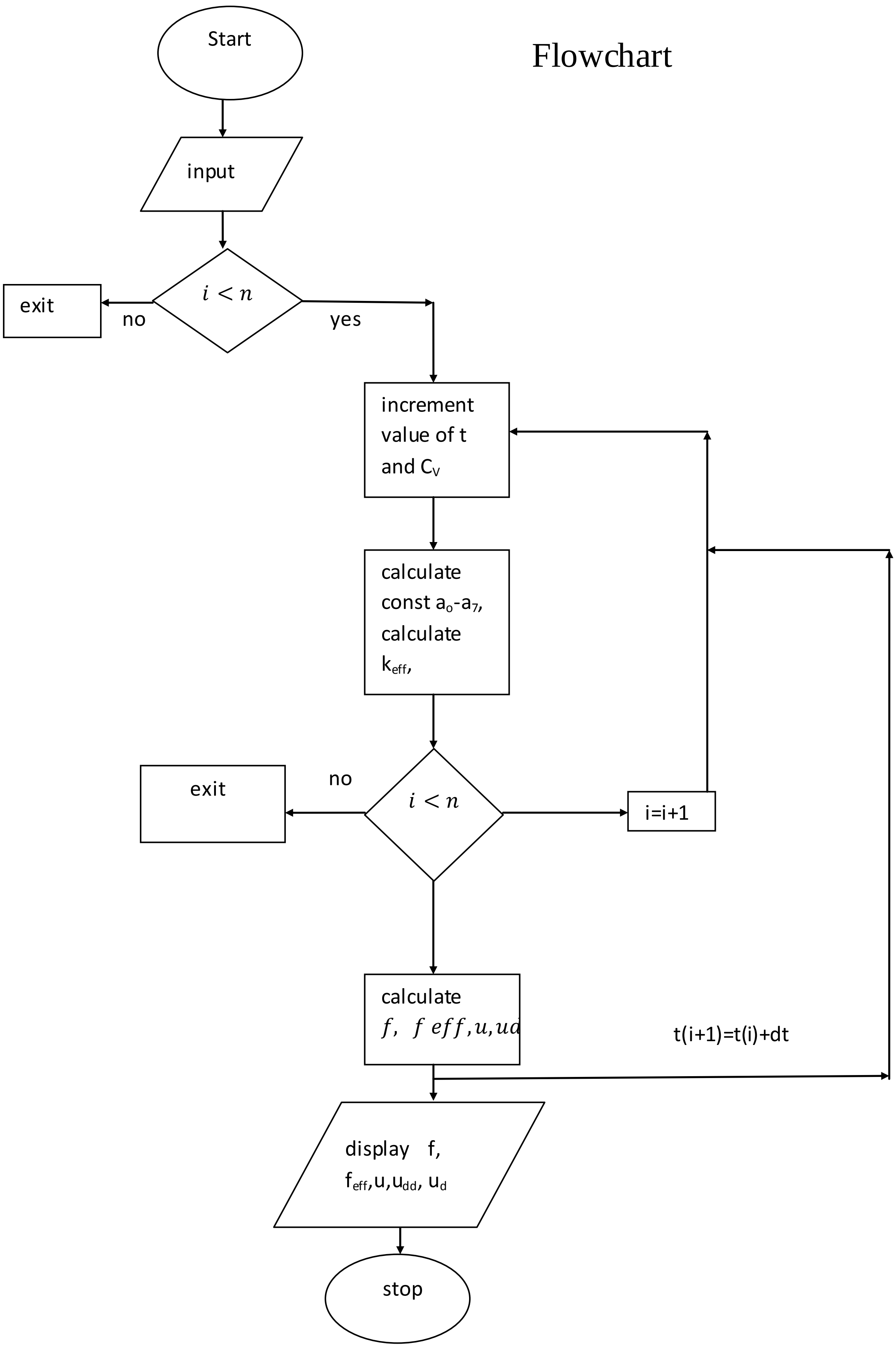
```
Input:
```

stiffness matrix=[3950000000 -3950000000 0 0;-3950000000 3950000000 0 0;0 0 6666.66 3333.33;0 0 3333.33 6666.66];
mass matrix=[6393.160 3196.630 0 0;3196.630 6393.260 0 0; 0 0 3.16*10^-3 1.58*10^-3; 0 0 1.58*10^-3 3.16*10^-3];
damping matrix c=[30 -30 0 0;-30 30 0 0 ;0 0 30 -30;0 0 -30 30];

b.2 For u-ϕ-P formulation

```
function [u,ud,udd] = Water Hammer( )
format long
n=input('no. of time increments required n=');
dt=input('time interval d t =');
t(1)=0;
for i=1:n
t(i+1)=t(i)+dt;
Cv=1.78*(i*dt)^0.239;
end
disp('input the stiffness and mass matrices');
k=input('stiffness matrix=');
m=input('mass matrix=');
c=input('damping matrix c=');
disp('stiffness matrix k=');
disp(k);
disp('mass matrix m=');
disp(m);
disp('damping matrix c=');
disp(c);
u=[0;0;0;1;1;1;0;0;0];
ud=[0;0;0;0;0;0;0;0;0];
udd=[0;0;0;0;0;0;0;0;0];
alpha=input('integration constantl alpha =');
gamma=input('integration constant gamma=');
a0=1/(alpha*dt^2);
a1=gamma/(alpha*dt);a2=1/(alpha*dt);
a3=(1/(2*alpha))-1;a4=(gamma/alpha)-1;a5=((gamma/alpha)-2)*(dt/2);
a6=dt*(1-gamma);a7=gamma*dt;
keff=k+a0*m+a1*c;
for i=1:n
f=[94.7 ;104.76; -199.95;190; 210; -400; 1.404*Cv; 51.63*Cv;-64.10*Cv];
disp('force matrix f=');
disp(f);
s = m*(a0*u+a2*ud+a3*udd)+c*(a1*u+a4*ud+a5*udd);
feff=f+s;
disp('feffective ::')
disp(feff)
temp1=u;
u=inv(keff)*feff;
temp=udd;
udd=a0*(u-temp1)-a2*(ud)-a3*(udd);
ud=ud+a6*temp+a7*udd;
disp('upadated value of u:')
disp(u)
% plot(t(i)+dt,u(i));
disp('upadated value of ud:')
disp(ud)
disp('upadated value of udd:')
disp(udd)
end
end
```

Flowchart



Start
input
$i < n$
exit
no
yes
increment value of t and C_V
calculate const a_o-a_7, calculate k_{eff},
exit
no
$i < n$
i=i+1
calculate $f,\ f\ eff, u, ud$
t(i+1)=t(i)+dt
display f, f_{eff},u,u_{dd}, u_d
stop